A Model for Human-Human Collaborative Object Manipulation and Its Application to Human-Robot Interaction

Ehsan Noohi, Miloš Žefran and James L. Patton

Abstract—During collaborative object manipulation, the interaction forces provide a communication channel through which humans coordinate their actions. In order for the robots to engage in physical collaboration with humans, it is necessary to understand this coordination process. Unfortunately, there is no intrinsic way to define the interaction forces. In this work, we propose a model that allows us to compute the interaction force during a dyadic cooperative object manipulation task. The model is derived directly from the existing theories on human arm movements. The results of a user study with 22 human subjects prove the validity of the proposed model. The model is then embedded in a control strategy that enables the robot to engage in a cooperative task with a human. The performance evaluation of the controller through simulation shows that the control strategy is a promising candidate for a cooperative human-robot interaction.

Index Terms—Physical human-robot interaction (pHRI), human-human cooperation, interaction force, dyadic object manipulation, cooperation strategy.

I. INTRODUCTION

MANY robotic applications require proactive physical interaction between a human and a robot. For instance, consider a robotic caregiver in the elderly care application. It has been shown that home care aides are most effective when they actively involve the elderly in physical activities [1], [2]. That is, it is important that the caregiver does not simply perform the task for the elder person, the elderly should be asked to proactively contribute in performing the task. Cooperative object manipulation (e.g. moving a table) is another example where proactive interaction is necessary. In general, in cases of cooperative physical interaction, successful completion of the task requires a coordinated force-exchange between the human and the robot.

While the human can usually amplify the cooperation performance by adjusting to the robot’s actions, in many applications the human is not capable of such an adjustment. For instance, in elderly care, rehabilitation, or childcare, the human is either an elderly person, a physically challenged patient or a toddler. In such applications, the human is not expected to adjust to the robot. Instead, the robot’s actions need to be as natural to the human as possible. Therefore, modeling the characteristics of natural human movements and the properties of the exchanged forces would significantly contribute to designing better robot control strategies in such applications.

In order to build a model for a natural (human-like) interaction, researchers have studied human-human collaborative tasks and proposed different models for the exchanged forces between humans. However, due to the physics of the task, there is an inherent ambiguity in trying to recover the interaction forces from the forces exerted by the humans. In other words, since the interaction force (the portion of the applied force that does not contribute to the motion of the object) is unknown, obtaining the effective portion of the applied force for each individual is challenging. To resolve this ambiguity, different models for the interaction force have been proposed [3]–[7]. In all of these models (including ours) the ambiguity is resolved by introducing additional constraints in the interaction model. For instance, Williams and Khatib [3] suggest that the interaction force follows the mechanical internal force. Groten et al. [4] assume that the human minimizes the energy of the interaction force during the collaboration. However, some of the suggested constraints can not be used in a cooperative collaboration setting. The leader/follower scheme [5] is another example of such constraints. Other models have also been proposed, but they are not informative in the physical human-robot interaction (pHRI) setup, as they only provide a descriptive model for an existing interaction (e.g. [6]). We will discuss the properties of these models in more detail in Sec. III-D.

The existing models compute the interaction force for every isolated pair of force vectors and ignore the time dependencies among the force pairs during the task. In contrast, our approach considers the whole trajectories of the force vectors to compute the interaction force. More specifically, the interaction force is obtained by exploiting a computational motion model of the nominal movement trajectory during the cooperation. In fact, the knowledge of the motion model serves as the constraint that resolves the ambiguity.

Our contribution is threefold: First, by exploiting the motion model associated with the task, we propose a descriptive model for the interaction force. The key advantage of the proposed model compared to other models is that it only requires one of the applied forces to be measured in order to compute the interaction force. As a result, during a human-robot collaborative task, the robot can model the interaction...
force by measuring only the force applied by the human. In turn, this significantly simplifies the robot’s controller. Details are given in Sec. IV.

Second, we propose a controller for the robot that exploits the interaction force model to determine the force that the robot should apply. While the robot is aware of the task, the human’s desired trajectory is unknown to the robot. Using the interaction force information, the controller responds to the human’s applied force in such a way that the human’s desired trajectory can be followed. We refer to this controller as offline controller, because the interaction force is computed offline. In Sec. VII, we statistically analyze the performance of the controller with and without the interaction force information. We show that the performance is statistically significantly higher when the interaction force information is provided.

Finally, we propose an online controller for the robot that exploits a real-time estimation of the interaction force. The estimation algorithm is based on our proposed model. It also utilizes a predication of the human’s future actions. In Sec. VIII, we introduce the online controller in detail and compare it with the offline controller. We show that no statistically significant difference exists in the performance of the two controllers.

The rest of the paper is organized as follows. In Sec. II, we discuss the relevant literature on the models for the human-arm movement and on the human-robot interaction. In Sec. III, we review the models of the arm movement that will be used in the paper and introduce the necessary background on modeling forces during cooperative manipulation. Our model for the interaction force is introduced in Sec. IV. To evaluate the model, we conducted a human study with 22 subjects. The study considers both dyadic manipulation (two-person) and single-person bimanual manipulation. The experimental setup is described in Sec. V. Using the collected data, we compare the performance of our model with two existing descriptive models in Sec. VI. In particular, we show that the proposed model can distinguish between the dyadic manipulation and the bimanual manipulation. In Sec. VII, we investigate the effect of the interaction force on the performance of the robot. We show that providing the robot with the interaction force (computed offline) significantly improves its performance. By further exploiting the properties of our model, we construct an interaction force estimator in Sec. VIII. The estimator is then incorporated into an impedance controller that implements the cooperation strategy for the robot. Finally, we present the performance evaluation that demonstrates the effectiveness of the proposed controller.

II. RELATED WORK

Human movement has been studied extensively. While formulating a model that accounts for trial-to-trial variations of the motion trajectory is a challenging research problem, the average motion trajectory has been successfully described with several models. It has been shown that many biological movement profiles exhibit certain regularities. In a large class of complex movements, e.g. drawing a curved path, Lacquaniti et al. [8] showed that the angular velocity and the path curvature are in a power relation, referred to as the 2/3rd power law. In grasping and reaching movements, Fitts [9] showed that a linear relation exists between the movement time and the index of difficulty (defined in terms of the distance to the target and the size of the target).

A number of successful models for biological movements have been proposed that are based on optimal control [10]. These models suggest that the human sensorimotor system optimizes a certain cost function in order to perform specific movements. Anderson and Pandy [11] showed that metabolic energy minimization reproduces the salient features of normal gait (for walking on level ground). Flash and Hogan [12] showed that a minimum-jerk model accurately describes trajectories of the reaching movements. This model generates the well-known bell-shape velocity profile and can describe the relation between the velocity and the curvature even more accurately than the 2/3rd power law [10]. The minimum torque change (MTC) model has been proposed as an alternative to the minimum-jerk model [13], [14]. In this model, the nonlinear dynamics of the arm is considered and the motion trajectory is obtained by minimizing the variations of the applied joint torques. The accuracy of the final position (with respect to the target point) is proposed as another optimization criterion. Harris and Wolpert [15] suggested that the optimal trajectory minimizes the final position variance, also known as minimum variance model. The model is shown to be very successful in describing speed profiles of saccadic eye movements and is consistent with both Fitts’ law and the power law.

While there exists a rich literature on modeling single arm reaching movements, bimanual and dyadic movements are less studied. Tresilian and Stelmach [16] showed that the aperture and transport components of a single-arm reach-to-grasp task is very similar to the bimanual performance of the same task. Garvin et al. [17] studied the bimanual reaching movement and proposed an extension to the minimum-jerk model by incorporating the rotational jerk of the object into the optimization criterion. Diedrichsen [18] studied bimanual reaching movement in one-cursor and two-cursor conditions and showed that the change in the control and in the adaptation are both optimal and task-dependent. Noohi and Žefran [19] have recently shown that the object’s motion trajectory during the bimanual and dyadic reaching movements is highly correlated with the minimum-jerk trajectory.

In addition to the motion trajectory, the cooperation aspect of dyadic and bimanual movements has been studied as well. The exchanged forces have been shown to convey rich information about the cooperation [20]. Many researchers have studied the haptic clues as a communication channel that facilitates the physical interaction. Guiard [21] suggested that in bimanual tasks, while one arm performs the majority of the workload, the other arm is responsible for fine tuning and corrections. Reed et al. [22] observed similar arm specialization in a dyadic task where one person is contributing more to the acceleration and the other person...
to the deceleration. Noohi and Žefran [23] characterized the dyadic object manipulation task, proposed a set of measures that evaluate the performance of haptically-coupled subjects and cross-validated those metrics with the subjects’ self-assessments. Groten et al. [24] suggested that in a shared decision making situation, the haptic channel enhances the intention integration and results in a higher performance. Ganesh et al. [25] reported that haptically-coupled subjects demonstrate better motor performance than a single person in a virtual pursuit-tracking task. Van der Wel textitet al. [26] show that in an object manipulation task, dyads amplify their applied forces to develop a haptic communication channel. Mörtl et al. [27] studied the process of role assignment (leader/follower roles) during the physical cooperation. Feth et al. [28] studied a joint-pursuit-tracking task and observed that the performance of a dyadic collaboration is higher than for single individuals. Based on the asymmetry of energy flow between the subjects, they suggested that the increased performance was due to the emergence of different roles. Sawers and Ting [29] have suggested that the increased performance was due to the fact that both hands receive synchronized commands from a common controller (person’s brain). Bilateral-training methods in rehabilitation exploit this feature and significant improvements in patients with post-stroke hemiparesis have been reported [30].

Many of these models have been adopted by researchers in assistive and rehabilitation applications in order to improve the performance of the physical interaction between a robot and a human. For instance, Corteville et al. [30] proposed an admittance control scheme for fast point-to-point cooperative sliding of an object. The controller takes advantage of an extended Kalman filter to estimate the parameters of the motion velocity. The velocity profile was assumed to follow the minimum-jerk model. Bussy et al. [31] proposed motion primitives for cooperative transportation of heavy objects and introduced a velocity-based algorithm to generate a sequence of such proposed primitive motions. Medina et al. [32] proposed a risk-sensitive optimal controller that responds to the uncertainty in the predicted human motion by adapting the robot’s role. They modeled the human as a process noise that the robot needs to consider in its optimization cost function.

III. BACKGROUND

A. Motion Trajectory during Reaching Movement

Reaching movement is one of the well-studied human motions. In this motion, a person moves his/her hand from point A to point B on a straight line. The motion is fast and happens over a short distance. When performing the single arm reaching movement, it has been shown that humans generate a smooth trajectory with the well-known bell-shaped curve for hand velocity [12]. More specifically, the hand trajectory minimizes the following cost function:

\[
H(t_i, t_f, x) = \frac{1}{2} \int_{t_i}^{t_f} \left( \frac{d^3 x(t)}{dt^3} \right)^2 dt
\]

\[
x^*(t) = \arg \min_{x(t)} (H(t_i, t_f, x))
\]

where \(x(t)\) is the position trajectory of the hand, \(t_i\) is the start time of the motion and \(t_f\) is the end time. If we take the assumption that the hand is at rest at the start and at the end of the reaching movement (i.e. zero boundary conditions), the minimum of the cost function would be: \(H^* = 360L^2\), and the optimal trajectory is: \(x^*(t) = x_0 + L(\tau^3 - 15\tau^4 + 10\tau^5)\). Here, \(\tau = (t - t_i)/(t_f - t_i)\) and \(L = ||x_f - x_i||\). Also, \(x_i = x(t_i)\) and \(x_f = x(t_f)\) are the positions of the hand at the start point and the end point, respectively. Since the cost function is the sum of the squared jerk along the motion trajectory, this model is referred to as the minimum-jerk trajectory hereafter.

B. Motion Trajectory of Dyadic Reaching Movement

In a bimanual object manipulation, the object is grasped by two hands and is moved from a starting point to an ending point. In single-person-bimanual manipulation (where both hands belong to the same person), it has been shown that the object closely follows an optimal trajectory [17]. That’s due to the fact that both hands receive synchronized commands from a common controller (person’s brain). Bilateral-training methods in rehabilitation exploit this feature and significant improvements in patients with post-stroke hemiparesis have been reported [33].

In a dyadic reaching movement however, the imperfect synchronization between the two cooperating persons can introduce deviation from the optimal trajectory. That is, although the motion’s start point \((x_i)\) and end point \((x_f)\) are known, the persons’ estimation/control of their hands’ positions are different. More importantly, each person has his/her own preferred speed. We have recently shown that the object’s motion trajectory during the dyadic reaching movement is highly correlated with the minimum-jerk trajectory [19]. More specifically, the Pearson correlation coefficient between the normalized object velocity and the velocity profile of a minimum-jerk trajectory is \(\rho = 0.97\) with \(CI_{95\%} = [0.95, 0.98]\).

Fig. 1 shows the 2D histogram of normalized object velocities for bimanual and dyadic object manipulation during a reaching movement task. The color of each bin represents the percentage of the samples observed in that bin.
Communicate with the other person [26], [38]. It can compress force can be represented as a 5th order polynomial:

\[ F = k_s(x(t) - x_0), \]

where \( k_s \) is the stiffness of the spring and, \( x_0 \) is the position of the spring’s end when no force is applied to it. After the adaptation period, the subject learns to cancel the force field and returns to their original trajectories [34]–[36]. Consider the situation, where a jerk trajectory. Since \( d^3x(t)/dt^3 = (-k_s^{-1}) d^3F(t)/dr^3 \), we can rewrite (1) as:

\[ F^*(t) = \text{argmin}_{F} \left( \frac{1}{2} \int_t^{t_f} \left\| \frac{d^3F(t)}{dr^3} \right\|^2 dr \right) \]  

(2)

Eq. (2) states that, in a spring-like force field, the applied force minimizes the squared-jerk cost function. Applying the calculus of variations technique on (2), it is easy to show that the 6th derivative of \( F^*(t) \) is zero [37]. Therefore, the applied force can be represented as a 5th order polynomial:

\[ F^*(t) = \sum_{k=0}^{5} c_k t^k \]  

(3)

Note that, if the hand is at rest at the start and at the end of the reaching movement, the applied forces are, too. In the general case that the hand is not at rest, to determine the \( c_k \) coefficients in (3), the minimization problem of (2) should satisfy the boundary conditions.

C. Applied Force during Reaching Movement

Disturbing the reaching movement by an external force field will deviate the hand trajectory from its optimal trajectory. It has been shown that, after enough learning trials, humans can adapt to the external force field and return to their original trajectories [34]–[36]. Consider the situation, where a subject performs a reaching movement while a spring resists his/her forces, see Fig. 2. The spring introduces the position-dependent force field \( F = -k_s(x(t) - x_0) \), where \( k_s \) is the stiffness of the spring and, \( x_0 \) is the position of the spring’s end when no force is applied to it. After the adaptation period, the subject learns to cancel the force field and returns to the optimal trajectory, \( x^*(t) \), for the reaching movement [35]. More interestingly, the force that the subject needs to apply at the first stage (to cancel the force field) is \( F^*(t) = -k_s(x^*(t) - x_0) \). In other words, the applied force follows a smooth minimum-jerk trajectory. Since \( d^3x(t)/dt^3 = (-k_s^{-1}) d^3F(t)/dr^3 \), we can rewrite (1) as:

\[ F^*(t) = \text{argmin}_{F} \left( \frac{1}{2} \int_t^{t_f} \left\| \frac{d^3F(t)}{dr^3} \right\|^2 dr \right) \]  

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D. Models for the Interaction Force

Consider a dyadic object manipulation task. Let \( f_1 \) and \( f_2 \) refer to the forces that are applied to the manipulated object and \( F_{sum} = f_1 + f_2 \) be the resultant force that is associated with the task. Each applied force can be decomposed into the effective force (\( f_1^* \) and \( f_2^* \)) and the interaction force (\( F^i \)) as:

\[ f_1 = f_1^* + F^i \]

\[ f_2 = f_2^* - F^i \]  

(4)

The interaction force can be used to secure the grasp or to communicate with the other person [26], [38]. It can compress or stretch the object, but it does not influence the object’s equations of motion. As a result, all force components that lie in the null space of \( F_{sum} \) (orthogonal to it) are part of interaction force. That is:

\[ f_1^* = \alpha F_{sum} \]

\[ f_2^* = (1 - \alpha)F_{sum} \]  

and therefore,

\[ F^i = (1 - \alpha)f_1 - \alpha f_2 \]  

(5)

(6)

where \( \alpha \) denotes the contribution of each person in performing the task.

Note that (4) is an under-determined system of equations and, any arbitrary value of \( \alpha \), \( 0 \leq \alpha \leq 1 \) introduces a valid decomposition. According to (5) and (6), the only situation in which the system has a unique solution is when \( F_{sum} = 0 \). In such instances, \( F^i = f_1 = -f_2 \) and \( f_1^* = f_2^* = 0 \). In all other situations, to be able to uniquely determine the interaction force, one needs to introduce an additional constraint to the system (e.g. introducing specific values for \( \alpha \)). Note that, we take the general case where both hands can apply pure torque to the object and thus, the direction of the interaction force is not necessarily aligned with the grasp configuration (no additional torque constraints).

Fig. 3 shows two possible decomposition examples for a single pair of applied forces. Note that in Fig 3b, the interaction force has both orthogonal and parallel components (w.r.t. \( F_{sum} \)). While obtaining the orthogonal component is straightforward, finding a computational model for the parallel component is challenging. That is the reason why we only focus on the parallel components of \( f_1 \), \( f_2 \) and \( F^i \) in this work. We will refer to these parallel components as the applied forces (\( f_1 \) and \( f_2 \)) and interaction force (\( F^i \)), hereafter. After the interaction force is computed, it will be augmented with the orthogonal component.

To uniquely determine the interaction force, researchers have considered different approaches. In case of the human-robot collaboration, some researchers tackle this problem by assigning the leader/follower roles to the human and the robot (\( \alpha = 0 \) or \( \alpha = 1 \)), e.g. [5]. Mörtl et al. [27] studied the exchange of the leader/follower roles between the human and the robot and observed that dynamic role assignments.
resulted in a considerably larger interaction force. Evrard and Kheddar [6] suggested that a continuous homotopy switching happens between the leader and the follower roles during the collaboration (for each agent independently). However, they did not identify the homotopy function for dyadic tasks. Leader/follower schemes are very successful in the tasks that the human steers the task and the robot provides the whole workload and follows the human.

Unlike leader/follower schemes, in this work we are interested in cooperative object manipulation task where both human and robot are actively contributing in performing the task. One of the most appreciated approaches is to take the mechanical internal force as the interaction force. For instance, while proposing a hybrid position/force control scheme for two coordinated robots, Uchiyama and Dauchez [39] described the interaction between the robots by the internal force/moment vector. Using this vector, the authors manage to describe the object deformation (including twisting, bending and shearing) during manipulation. Williams and Khatib [3] introduced a more general framework to properly design the interaction forces for multi-contact robot-robot collaborative tasks, namely “virtual linkage” model. They suggested that an acceptable interaction force would minimize the engineering strain (minimum deformation of the object). They used the internal forces as the interaction force in multi-robot object manipulation. In case of two robots, the force decomposition would become:

\[
\begin{align*}
F_i' &= f_1^* + f_2^* = \frac{1}{2}F_{\text{sum}} \\
F_t &= \frac{1}{2}(f_1 - f_2)
\end{align*}
\]

or \( \alpha = 0.5 \). Taking the difference of the applied forces as the interaction force, as expressed in (7), is a common practice in the human-robot interaction literature as well, e.g. [7].

Another interesting assumption in the literature, proposed by Groten et al. [4], is that human minimizes the energy of the interaction force during the collaboration. They studied the dominance distribution in a dyadic haptic collaboration and implicitly exploited the minimum-energy model. The assumption is based on the intuition that when only one force is applied to an object, the interaction force should be zero. Fig. 4 illustrates this intuitive assumption in comparison with the virtual linkage model. This model finds a solution for (5) and (6) in which the magnitude of the interaction force is minimum. In other words:

\[
\alpha(t) = \arg\min_{\alpha} (||F'||)
\]

Fig. 5a shows a typical example of a bimanual object manipulation in 1-D. The interaction force before the start of the motion \( (t \leq t_f) \) and after the end of the motion \( (t \geq t_f) \) is \( F_i' = 2 \), which is the grasp force to hold the object. Fig. 5b shows the interaction force and the effective forces obtained from virtual linkage model. Here, \( F_i' = 2 \) and \( f_1^* = f_2^* = \frac{1}{2}F_{\text{sum}} \). The model suggest a perfect load-sharing cooperation, which is a likely behavior in a single-person-bimanual manipulation. Fig. 5c depicts the interaction force and the effective forces based on the minimum-energy assumption. Under this assumption, the first (second) hand applies the whole required force \( (F_{\text{sum}}) \) to move the object in the acceleration (deceleration) phase of the manipulation, while the other hand applies zero force. Taking the accelerating/decelerating roles is an expected behavior in dyadic collaborative tasks [22]. We will compare our proposed model with these two models in the following sections.

IV. PROPOSED MODEL FOR THE INTERACTION FORCE

In this section, we first compute the interaction force in a mass-spring system. Then, we discuss how a mass-spring system generalizes a rigid body model for a solid body. Next, we propose our model for the interaction force, using the knowledge of the task. Finally, we discuss different aspects of the proposed model and list its key features.

A. Interaction Force in a Mass-Spring System

Consider the cooperative manipulation of the mass-spring system in Fig. 6. The masses \( m_1 \) and \( m_2 \) are coupled with a spring and the forces \( f_1 \) and \( f_2 \) are applied to them, respectively. The positions of the masses \( m_1 \) and \( m_2 \) are referred to by \( x_1 \) and \( x_2 \) \( (x_1 > x_2) \), respectively. The task is to move the masses from their initial configurations \( (x_{i1} \) and \( x_{i2} \) ) to the goal configurations \( (x_{g1} \) and \( x_{g2} \) ) on a straight line (aligned with the x-axis).

If the position variables are available (or can be measured), the interaction force that the spring introduces to the system is computed as:

\[
F_i'(t) = -k_s(x_1(t) - x_2(t) - L_0)
\]
where $k_s$ is the stiffness of the spring and $L_0$ is the length of the spring when no force is applied to it.

On the other hand, if $x_1$ and $x_2$ are not available but the applied forces $f_1$ and $f_2$ are, then the interaction force would be computed from the equations of motion. For the above applied forces $f_i$ and $f_2$, the equations of motion can be expressed as:

$$m_1 \ddot{x}_1(t) = f_1(t) - F^i(t)$$

$$m_2 \ddot{x}_2(t) = f_2(t) + F^i(t)$$

(10)

By combining the equations in (10), we will have:

$$m_1 m_2 (\ddot{x}_1 - \ddot{x}_2) = m_2 f_1 - m_1 f_2 - (m_1 + m_2) F^i$$

(11)

Taking the second derivative of (9) would result in:

$$F^i(t) = -k_s (x_1(t) - \ddot{x}_2(t))$$

(12)

And plugging (12) into (11) would introduce the interaction force only in terms of the applied forces:

$$C \ddot{F}^i(t) + F^i(t) = (1 - \alpha) f_1(t) - \alpha f_2(t)$$

(13)

where

$$C = -\frac{m_1 m_2}{k_s (m_1 + m_2)} \quad \text{and} \quad \alpha = \frac{m_1}{m_1 + m_2}$$

(14)

Equations (13) and (14) describe the interaction force in the form of a dynamical equation when $f_1$, $f_2$, and the system parameters ($m_1$, $m_2$ and $k_s$) are available.

B. Mass-Spring System vs. Rigid Body

Modeling objects as a rigid body is a common practice in many problems, due to its simplicity and effectiveness. It is based on the fact that deformation of a solid body in response to small (or moderate) forces is negligible for many materials. This is especially true when the applied forces are within the range of a human-human cooperative manipulation. However, using the rigid body model and ignoring the deformation of the solid body is in part the source of the ambiguity in computation of the interaction force (as discussed in Sec. III-D).

The mass-spring system generalizes the rigid body model by including object deformation in the model (the spring). For instance, if we replace the spring with a massless rigid rod (or equivalently $k_s \to \infty$ in the spring), the mass-spring system becomes one rigid object (with two separated masses). Since the rod is rigid, we have $x_1 = x_2 + L_0$ and therefore, (11) becomes the same as (6), where $\alpha$ is defined in (14).

Although the generalization introduced by mass-spring system seems promising, unfortunately, neither (9) nor (13) can be used to compute the interaction force in a solid body. In case of (9), although the position variables ($x_1$ and $x_2$) are assumed to be available, accurate measurement of these variables is practically infeasible. Furthermore, the value of $k_s$ in a solid body is very large. As a result, any small noise in the measurement of $x_1$ and $x_2$ would result in a large error in the computation of $F^i$.

Similarly, exploiting (13) to compute the interaction force in a solid body is impractical. Here, the large value of $k_s$ would not be a problem and, in fact, it results in a negligible value for $C$ in (14) and thus, would transform (13) to be the same as (6), as expected. The problem is that any arbitrary values for $m_1$ and $m_2$ would work in the mass-spring system and generate a valid model for the solid body. For instance, if $m_1 = m_2$, we get $\alpha = 0.5$ in (14) and it would represent the virtual linkage model. Or similarly, when $m_1 = 0$ ($\alpha = 0$), the mass-spring system represents the leader/follower model in which $f_1$ leads the task. In other words, the mass-spring system cannot disambiguate the interaction force model in (6), if (13) is being exploited.

In contrast, we will show that by including the information about the task, we can tackle both aforementioned problems. That is, the proposed model for the interaction force is both non-ambiguous and robust to the measurement noise. To tackle the ambiguity issue, it implicitly takes advantage of the position variables. And to address the noise problem, it exploits the measurements of the applied forces.

C. Task-Aware Interaction Model: A Polynomial Model

Let us assume that the mass-spring system is being manipulated cooperatively, in a cooperative reaching movement task. That is, the subjects move the masses from their initial configurations ($x_{i_1}$ and $x_{i_2}$) to their final configurations ($x_{f_1}$ and $x_{f_2}$), following a minimum-jerk trajectory. Since each hand experiences disturbing forces through the spring (the interaction force), the reaching movement is performed inside a force field. As discussed in Sec. III-C, after enough learning trials, cooperating hands learn to compensate for the force field and return to their original minimum-jerk motion trajectories:

$$x_1(\tau) = x_{i_1} + (x_{f_1} - x_{i_1})(6\tau^5 - 15\tau^4 + 10\tau^3)$$

$$x_2(\tau) = x_{i_2} + (x_{f_2} - x_{i_2})(6\tau^5 - 15\tau^4 + 10\tau^3)$$

(15)

where $\tau = (t - t_f)/(t_i - t_f)$ and $t_i$ and $t_f$ are the start time and the end time of the cooperative manipulation. As a result, $\Delta x(t) = x_1(t) - x_2(t)$ would also be a minimum-jerk trajectory:

$$\Delta x = \delta_1 + \left(\delta_2 - \delta_1\right)(6\tau^5 - 15\tau^4 + 10\tau^3)$$

(16)

where $\delta_1 = (x_{i_1} - x_{i_2})$ and $\delta_2 = (x_{f_1} - x_{f_2})$. Let us rewrite (9) as $F^i(t) = -k_s (\Delta x(t) - L_0)$. Similar to the discussions in Sec. III-C, when $\Delta x(t)$ is a minimum-jerk trajectory, the interaction force would also be a minimum-jerk trajectory:

$$F^i(t) = \arg\min_f \left( \frac{1}{2} \int_{t_i}^{t_f} \left\| \frac{d^3 F(t)}{dr^3} \right\|^2 dr \right)$$

(17)
or equivalently, the interaction force can be expressed as:

$$F^i(t) = \sum_{k=0}^{5} c_k t^k$$  \hspace{1cm} (18)

For a complete model, we need to determine the coefficients, $c_k$. As explained in Sec. III-D, when $F_{\text{sum}} = 0$, the value of $F^i = f_1$ is known. According to Sec. III-B, $F_{\text{sum}}$ is associated with a minimum-jerk trajectory (bell-shaped velocity profile). That is, it has exactly one zero crossing point, namely $t_m$. Therefore, $F_{\text{sum}} = 0$ for $t \leq t_i$, $t = t_m$ and $t \geq t_f$, see Fig. 5a. This constructs three constraints for the interaction force:

$$
\begin{align*}
F^i(t_i) &= f_1(t_i) \\
F^i(t_m) &= f_1(t_m) \\
F^i(t_f) &= f_1(t_f)
\end{align*}
$$  \hspace{1cm} (19)

Since $F^i(t) = f_1(t)$ for $t \leq t_i$ and $t \geq t_f$, we will have $\dot{F}^i(t_i^-) = \dot{f}_1(t_i^-)$ and $\dot{F}^i(t_f^+) = \dot{f}_1(t_f^+)$. And since both signals are smooth signals:

$$
\begin{align*}
F^i(t_i) &= \dot{f}_1(t_i) \\
F^i(t_f) &= \dot{f}_1(t_f)
\end{align*}
$$  \hspace{1cm} (20)

The five constraints in (19) and (20) determine five coefficients in (18) and the last coefficient is obtained by solving the optimization problem in (17). To calculate the coefficients, let $P_k(t)$ denote the 4th order polynomial that satisfies the constraints in (19) and (20), i.e.:

$$
\begin{align*}
P_k(t) &= e_0 t^4 + e_1 t^3 + e_2 t^2 + e_3 t + e_4 \\
P_k(t_i) &= f_1(t_i) \\
P_k(t_m) &= f_1(t_m) \\
P_k(t_f) &= f_1(t_f)
\end{align*}
$$  \hspace{1cm} (21)

Thus, any 5th order polynomial that satisfies the constraints in (19) and (20) can be expressed as:

$$P_5(t, \kappa) = P_5(t) + \kappa(t-t_i)^2(t-t_m)(t-t_f)^2$$  \hspace{1cm} (22)

The interaction force is the 5th order polynomial that satisfies (17). Thus, by solving (17) for (22), the optimal value of $\kappa$ is obtained as:

$$
\kappa^* = \arg\min_{\kappa} \left( \frac{1}{2} \int_{t_i}^{t_f} \left\| \frac{d^3 P_5(t, \kappa)}{d t^3} \right\|^2 dt \right)
$$  \hspace{1cm} (23)

And the interaction force will be:

$$F^i(t) = P_5(t, \kappa^*)$$  \hspace{1cm} (24)

Equations (21)-(24) provide the computational model for the interaction force during a dyadic reaching movement.

D. discussion

To better understand different aspects of the proposed model, we provide a list of the characteristics of the model:

1) Assumption: As discussed in Sec. III-D, to uniquely determine the interaction force, one needs to introduce a new constraint to the system. The proposed polynomial model assumes that the characteristics of the task is given in the form of a computational model for the motion profile (here the minimum-jerk trajectories). As a result of this assumption, the model does not depend on the value of the object’s stiffness, $k$, or the selection of $m_1$ and $m_2$.

2) Limitation: The proposed interaction force model is based on the model motion of the cooperative hands. As a result, the level of precision of the model is determined by the precision of the motion model. Since minimum-jerk robot needs to compute the interaction force in real-time. We will further exploit this in Sec. VIII.

3) Advantage: Unlike the interaction force models discussed in Sec. III-D, the polynomial model does not require the whole trajectory of applied forces ($f_1$ and $f_2$) to obtain the interaction force. More importantly, it can extract the interaction force based on only one of the applied forces, e.g. $f_1$ in (21). This is particularly important when the robot engages in a cooperative task with a human. We will further elaborate on this in Sec. VIII.

4) Disadvantage: The interaction force models discussed in Sec. III-D can compute the value of $F^i$ at time $t$ only by measuring $f_1$ and $f_2$ at the same time, $t$. The proposed model, however, depends on the values of $f_1$ in the future times (i.e. at boundary points). This would be a major issue when the robot needs to compute the interaction force in real-time. We will further discuss this in Sec. VIII and IX.

5) Extension: The mass-spring model relates the interaction force to the deformation of the object. Since elastic deformation appears in many different forms (such as compression, tension, shear and torsion), one might suggest a more general form of linear relation between the interaction force and the deformation strain. For instance, the model can simply be extended to a more general case by using a mass-spring-damper system. Taking $k_d$ as the damping coefficient, the interaction force would be expressed as $F^i(t) = -k_c (x_1(t) - x_2(t) - L_0) - k_d (\dot{x}_1(t) - \dot{x}_2(t))$. It is easy to see that, if the motion profiles of $x_1$ and $x_2$ are available, the interaction force would be obtained as a linear combination of these profiles and their derivatives. In case of the reaching movement, the resulting interaction force would again be a 5th order polynomial.

V. HUMAN STUDY

To be able to evaluate the proposed interaction model, we collected human interaction data for bimanual and dyadic object manipulation tasks. In single-person-bimanual (SPB) mode, the subjects were asked to grasp an object with both hands and move it horizontally. In dyadic mode, the subjects were grouped into pairs and asked to perform cooperative manipulation. In both cases, the grasps were prehensile and the subjects could apply independent forces and torques to the object.
The handles and, grasp force has dominant components in this direction of the motion is parallel with the line connecting the handles. Therefore, the grasp force has small components in the motion direction. In type 2 motions, the direction of the motion is perpendicular to the line connecting the handles. The blue arrow on the table shows the motion direction and the red double arrow is the grasp force.

Next, each subject was given enough familiarization trials (as much as they needed). Then, he/she performed three trials in SPB scenario, including a short-range type 1 motion, a short-range type 2 motion and a long-range type 2 motion. The long-range type 1 motion was skipped, because it was not within the range of human-arm reachable space. Then the subjects were grouped into pairs (dyadic mode). Subjects in each pair stood on the opposite long-sides of the table and performed three more trials. The motion direction in all trials is along the length of the table. To generate different motion types, the pot was yawed to be parallel with or orthogonal to the motion direction. In all of the trials, no repeated measurements were collected. Each session took less than an hour.
hour and each participant were paid upon completion of all of the tasks.

D. Data Analysis

We collected 33 trials (11 × 3) in dyadic mode and 63 trials (21 × 3) in bimanual mode (one of the participants refused to complete the task in this mode). Then, the collected data was analyzed to identify the measurement errors. The errors were mainly due to the hysteresis error that the sensors exhibited randomly. We examined the value of $F_{sum}$ to identify the measurement error. For instance, if the pot was not moving (according to the IMU readings), a constant large value in $F$ was observed. We used the IMU readings in this procedure for a single sample trial. The collected applied forces and computed interaction force are shown in panel (a) and the effective forces along with $F_{sum}$ are presented in panel (b). It is worth mentioning that the optimization problem in (23) was solved numerically, using the optimization toolbox in Matlab. The above data processing procedure was performed again, using the virtual linkage model (VL) and minimum energy model (ME). Here, $F^i$ was computed using (7) and (8), respectively.

VI. EVALUATION OF THE PROPOSED MODEL

In this section, we study the characteristics of the polynomial model (PM) in comparison with the virtual linkage model (VL) and the minimum energy model (ME). Since the models are based on different assumptions, it is expected that they are different in their interaction force signals, $F^i(t)$, show different properties. To explore different features of an interaction force signal, we use five different quantitative metrics (given in the Appendix). Different metrics measure different features of a signal, such as average, energy, variation rate and etc. Furthermore, since computing either of interaction force ($F^i$), effective forces ($f_1^*$ and $f_2^*$) or $\alpha$ gives the other two (refer to (4) and (5)), different metrics use different variables to explore the features space more broadly, see (33)-(37). For more details on the metrics and their properties, refer to [23].

Recall that, each subject performed three trials in each session. Also recall that for each trial, the interaction force was computed in three different ways (three models). Since for each computed force we use five features, a total of 45 observations is collected per subject. On the other hand, we had 32 subjects (21 participants and 11 pairs), and therefore, the full list of observations would have 32 × 45 entries. However, to deal with the seven missing data (computed measurements), we performed a listwise deletion and thus, the number of subjects became 16 participants and nine pairs. We grouped the subjects according to the cooperation mode. That is, 16 participants were placed in “Bimanual” group and nine pairs were placed in the “Dyadic” group.

We are interested in studying the effect of the cooperation mode on the properties of the interaction force. Since only one brain controls both hands in bimanual mode (vs. two brains in dyadic mode), it is expected that the properties of $F^i$ are different between the two modes. To study this hypothesis, we set up a repeated measures ANOVA test. The between-subjects factor (mode) has two levels, bimanual and dyadic. The within-subjects factors are model (three levels), task (three levels) and feature (five levels). The test includes a total of 45 repeated measurements for each of the 25 subjects. Mauchly’s test of sphericity showed that the assumption of sphericity had been violated ($\chi^2(989) > 10000, p < .0001$), and therefore, a Greenhouse-Geisser correction ($\hat{\epsilon} = 0.086578$) was applied. The repeated measures ANOVA with a Greenhouse-Geisser correction determined that a statistically significant interaction exists between the “cooperation mode” and the “interaction model” ($F(0.173, 3.983) = 187.21, p < 0.0019$). The significance level was $\alpha = 5\%$ in this test. No any other statistically significant interaction was observed ($p > 0.05$ for all). No significant main effect was observed, except for the cooperation mode ($F(0.173, 3.983) = 187.21, p < 0.0019$) which was shadowed by its significant interaction with “interaction model”.

A pairwise multiple comparisons post-hoc test with the Bonferroni correction was performed. We observed significant differences between the means of the PM, VL and ME, when the cooperation mode was bimanual ($p < 0.0001$ for all). In case of the dyadic cooperation, while significant difference between the mean of VL and the means of the PM and ME were observed ($p < 0.0001$ for both), no significant difference was observed between the means of PM and ME ($p > 0.88$). On the other hand, a significant difference between the means of “Bimanual” and “Dyadic” groups was observed in “PM” model ($p < 0.0001$), while no significant difference was observed in case of “VL” and “ME” models ($p > 0.76$).

The test results indicate that, regardless of the cooperation mode, VL model introduces a statistically significantly different $F^i$ than PM and ME. Furthermore, PM and ME introduce statistically significantly different $F^i$ in bimanual mode. However, no enough evidences existed to draw the same
important to note that we based our study on a specific set of metrics. While we did our best to explore different aspects of the interaction force, it is possible that this set has not captured all important dimensions of the feature space. As a result, the aforementioned conclusion is under the influence of the metrics defined in the Appendix.

VII. ROBOT CONTROLLER BASED ON THE INTERACTION FORCE

In this section, we test our hypothesis that the robot performs more efficiently when it is provided with the interaction force. We introduce a controller scheme that exploits the interaction force to generate robot’s applied force. The efficiency of the robot is evaluated, using a measure of position error.

A. Simulation Setup

To make sure that the measured error reflects the difference between the models (and not dominated by another factor), we take the following steps. We assume that the robot has a full knowledge about the task; that is, \( x_i, x_f, i_i, i_f \) and the object’s mass, \( M \), are all known to the controller. We chose the impedance control over position control to make the controller responsive to the interaction force. We also assumed that the robot can be controlled perfectly, so that the commanded force appears at the robot’s end-effector identically and immediately.

Since the controller is based on the offline computed interaction force, it is not causal. Particularly, it cannot be utilized for a real robot that is interacting with a human in an online fashion. Throughout this section, the term “robot” refers to a virtual agent (or the controller itself), rather than an actual robot. In Sec. VIII, we introduce a causal controller to be implemented on a real robot. The controller exploits an online estimate of the interaction force instead of the offline computed one.

Fig. 11 shows the block diagram for the offline controller. The contribution of the robot to the manipulation task is shaped by the “Motion Profile” block. It generates the desired object trajectory \( x_d \) that is a minimum-jerk trajectory in our case (see Sec. III-B). The “Impedance Control” block generates the controller force \( F_{ctrl} \), which guarantees a stable tracking of this desired trajectory. Thus, \( F_{ctrl} \) is the contribution of the robot to the cooperative task. On the other hand, the “Interaction Model” block provides the interaction force \( F_{int} \). Thus the robot applied force, \( F_R \), is then construed as:

\[
F_R = F_{ctrl} - F_{int} \tag{25}
\]

As illustrated in Fig. 10a, when PM model is employed, bimanual and dyadic modes are statistically significantly different in all metrics (except \( I_2 \)). However, when VL model is used, no discrepancy exists between the metrics, see Fig. 10b. That is due to the fact that \( f_i^* = f_j^* \) and \( \alpha = 0.5 \) when the VL model is employed (see (7)). Plugging these values in (33)-(37) would give a constant value of one for all metrics. Finally, Fig. 10c shows that the confidence intervals in bimanual and dyadic modes overlap (except for \( I_2 \)) when ME model is used.

The test results suggest an interesting conclusion: PM would effectively capture the difference between the bimanual mode and the dyadic mode, VL would always fail to do so and no enough evidence existed that ME can do the same. In other words, considering existing sample population, the proposed polynomial model captures the critical information of the interaction better than the other two models. However, it is
Based on (4), “Human-human” cooperation and Human-Robot cooperation can be formulated as:

\[
\begin{align*}
F_H &= F_H^i + F_i \\
F_h &= F_h^i + F_i \\
F_R &= \bar{F}_{ctrl} - F_{intrct}
\end{align*}
\] (26)

where \(F_H\) (\(F_h^i\)) and \(F_h\) (\(F_h^i\)) are the human applied (effective) forces. More specifically, \(F_h\) is the force that we intend to replace with the robot force (\(F_R\)) and \(F_H\) is the force that the robot wants to cooperate with.

If we take the calculated human interaction force as the human-robot interaction force, \(F_{intrct} = F^i\), and if the robot uses the Human-human motion trajectory \((x_{HH})\) as the desired trajectory, \(x_d = x_{HH}\), the robot’s effective force will follow the other human’s effective force, \(F_{ctrl} = F_h^i\). Here, we assume that the actual motion trajectory of a Human-human dyadic manipulation \((x_{HH})\) is not available to the robot. Thus, the robot uses the minimum-jerk model as an estimate of the Human-human motion trajectory and therefore, \(F_{ctrl}\) approximates \(F_h^i\). The impedance control that provides robot’s effective force is:

\[
F_{ctrl} = M \ddot{x}_d + K_d (\dot{x}_d - \dot{x}) + K_s (x_d - x) - \frac{1}{2} M \ddot{g}
\] (27)

where, \(M\) is the object’s mass, \(K_d = 5M\) and \(K_s = 6M\). The values of \(K_d\) and \(K_s\) are selected in such a way that the poles of the closed loop system are small (\(p_1 = -2\) and \(p_2 = -3\)). It allows the robot to interact with the human, while keeping the tracking error small. The last term in (27) compensates half of the weight of the object, assuming that the human compensates the other half. Let’s apply (25), (26) and (27) to the object’s equation of motion, \(F_R + F_H + M \ddot{g} = M \ddot{x}\). Therefore,

\[
M \ddot{e} + K_d \dot{e} + K_s e = \bar{F}_H
\] (28)

where, \(e = x - x_d\) and \(\bar{F}_H\) is the human’s gravity-compensated effective force, \(\bar{F}_H^i = \bar{F}_H - \frac{1}{2} M \ddot{g}\). Eq. (28) shows that the controller is BIBO stable and the tracking error is a function of human effective force.

B. Simulation Results

For each trial of the human study, we consider the applied forces to the pot \((f_1\) and \(f_2)\) and calculate the interaction forces \((F^i)\) based on three models: virtual linkage (VL), minimum energy (ME) and polynomial model (PM). These interaction forces are used in the “Interaction Model” block in the robot controller in Fig. 11. That is, we apply \(F_{intrct} = F^i\) for above three models. To study the necessity of including the interaction force in the robot controller, we consider the case where \(F_{intrct} = 0\). We will refer to this model by zero-interaction-force (ZF) model.

Fig. 12 illustrates an example of the performance of the robot for different interaction models. The motion trajectory of the Human-human cooperation \((x_{HH})\) is considered as the base line for comparing the performance of the models. In Fig. 12a, this trajectory is marked with small circles \((Hh)\). The motion trajectories of VL, ME and PM closely follow the human-human trajectory. However, the zero-interaction-force \((ZF)\) trajectory cannot follow \(x_{HH}\). This means that, although the impedance controller attempts to contribute to the task and move the pot along the desired trajectory, due to the lack of a good model for the interaction force, robot forces conflicts with human’s and the whole task fails. It is more evident in Fig. 12b, where the difference between the \(x_{HH}\) and the human-robot motion trajectories \((x_{HR})\) with different interaction models are illustrated. While the position error for VL, ME and PM are very similar and less than 5 cm, the position error for ZF increases to 28 cm. We will use this error signal to statistically compare these models with each other in the next section.

Fig. 12c shows the human applied force \((\bar{F}_H)\) and the interaction forces generated by different models. As illustrated here, VL and ME models generate very similar interaction...
forces and, PM generates a low-order approximation of those interaction forces. These interaction forces shape the robot’s applied force ($F_R$). As it appears in Fig. 12d, the robot’s force ($F_R$) follows the replaced human force ($F_h$) in all models, as expected. However, there exist a distinct difference between $F_R$ and $F_h$, due to the difference between robot’s desired trajectory and human’s original trajectory ($x_d \neq x_{Hh}$).

C. Statistical Evaluation

We are interested in studying the effect of different interaction force models (PM, VL, ME, ZF) on the performance of the proposed controller. Comparing the performance requires an evaluation measure to be introduced. Let $d(t)$ be the difference between the human-human motion trajectory and the human-robot motion trajectory, $d(t) = x_{Hh} - x_{HR}$. We used the root-mean-squared of $d(t)$ as the evaluation measure:

$$RMSE = \sqrt{\frac{1}{t_f-t_i} \int_{t_i}^{t_f} \|d(t)\|^2 dt} \quad (29)$$

The measure evaluates the performance of the controller in terms of the average position error, given the specific interaction model is used. We use the sample population of the $RMSE$ to statistically compare the effect of the interaction models. The mean and standard deviation of the $RMSE$ for different models are: PM: $(2.36 \pm 2.31)$, VL: $(2.27 \pm 2.35)$, ME: $(2.25 \pm 2.32)$ and ZF: $(9.43 \pm 9.09)$ all in cm. Fig. 13 shows the mean and standard deviation of RMSE measure, when different interaction models are employed. As it is illustrated in the figure, the ZF model generates larger mean error than other three models and, the average error introduced by PM is not very different from VL’s error or ME’s error.

In Sec. VI we discussed that the properties of the interaction force are different among different models. Therefore, it is expected that the performance of the controller would also be different between the models. To evaluate this hypothesis, we set up another repeated measures ANOVA test. The ANOVA model is very similar to the one described in Sec. VI. The between-subjects factor “mode” and the within-subjects factor “task” are the same as before. But, the within-subjects factor “model” has four levels (PM, VL, ME, ZF) here. The test includes a total of 12 ($3 \times 4$) repeated measurements for each of the 25 subjects.

Mauchly’s test of sphericity showed that the assumption of sphericity had been violated ($\chi^2(65) = 894.05, p < .0001$), and therefore, a Greenhouse-Geisser correction ($\hat{\epsilon} = 0.23046$) was applied. The repeated measures ANOVA with a Greenhouse-Geisser correction determined that the mean $RMSE$ differed statistically significantly among different models ($F(0.6914, 15.9017) = 25.576, p < 0.0001$). Furthermore, no statistically significant interaction was observed between “model” and other factors ($p > 0.15$ for all). The significance level was $\alpha = 5\%$ in the test. A pairwise multiple comparisons post-hoc test with the Bonferroni correction was performed. We observed that the mean RMSE error is statistically significantly higher when ZF is used ($p < 0.0003$ for all three pairwise tests). Moreover, no statistically significant difference was observed between the means of PM, VL and ME ($p > 0.82$ for all tests). The test results suggest the following two conclusions.

First, informing the controller about the properties of the interaction force would significantly increase its performance. This conclusion, which is not surprising, would support the exploitation of the interaction force models in designing an online controller (see next section).

Second, no enough evidence existed to suggest that the performances of different controllers (with PM, VL and ME models) are statistically significantly different from each other. This is a surprising conclusion, considering our discussion on the differences between the properties of the interaction force among these models (see Sec. VI). We speculate that, the impedance controller block would have partially compensated for the differences between the interaction models and resulted in this observation.

VIII. HUMAN-ROBOT COOPERATION STRATEGY

In designing the control scheme in the previous section, we discussed that taking the human-robot interaction force equal to the calculated human interaction force ($F_{intrct} = F^t$) leads to a human-robot motion trajectory that closely follows the human-human motion trajectory. Thus, we assumed that the interaction force between humans was fully known (calculated offline) and the “Interaction Model” block in Fig. 11 was designed in a feed-forward manner.

In case of a real-time human-robot interaction, no human-human interaction exists as a reference and therefore, the controller needs to calculate the interaction force only based on the human’s applied force $F_H$. Fig. 14 shows the real-time robot controller block diagram, in which the “Interaction Model” block measures the human force $F_H$ and estimates the interaction force $F_{intrct}$. The design of the rest of the controller is exactly the same as discussed in the previous section. In the next section, we introduce an algorithm that estimates $F_{intrct}$, by predicting $F_H$ and utilizing the polynomial model.

A. Estimating the Interaction Force

We showed that the polynomial model (PM) demonstrates a satisfactory performance in offline controller. We also discussed that, while other interaction models require both human
Forces ($F_H$ and $F_b$) to calculate the interaction force ($F^i$). PM provides a good approximation for $F^i$, using only one force ($F_H$). Moreover, the model requires only five boundary values to identify the interaction force. As a result, polynomial model is a good candidate for estimating $F^i$. According to the PM model, (21)-(24), the interaction force requires the value of $F_H$ to be known at times $t_i$, $t_m$ and $t_f$. However, when the robot is calculating $F_{intrct}(t)$ for times $t < t_m$ (or $t < t_f$), the future human force, $F_H(t_m)$ (or $F_H(t_f)$), is not available yet. Therefore, in order to use PM model as an estimator of the interaction force, either the boundary values or the whole future human force need to be predicted.

Let us define the predicted human force $F_H$ as:

$$F_H(\tau, t) = \begin{cases} F_H(\tau) & \tau \leq t \\ F_H(t) & \tau > t \end{cases}$$  \hspace{1cm} (30)

where $\tau$ is the time variable ($0 \leq \tau < \infty$) and $t$ represents the current time. The predictor suggests that human’s applied force in future remains constant and equal to the current force value. While it is the simplest prediction of the future values of $F_H$, it is motivated by the minimum-jerk constraint. It is easy to see that (1) minimizes $||F_H(t)||^2$ and thus, $F_H(t) = 0$ (or $F_H(t) = Const.$) is the optimal solution. However, this justification is based on the assumption that the manipulation task is mainly performed by the robot and human only provides a constant interaction force (e.g. grasp force).

If we plug (30) into (21)-(24), the interaction force that is predicted at time $t$ for the whole task ($t_i \leq \tau \leq t_f$) would be obtained as:

$$F^i(\tau, t) = \text{PolynomialModel}(F_H(\tau, t))$$  \hspace{1cm} (31)

which is a 5th order polynomial, satisfying the predicted boundary values. As a result, the estimated interaction force at time $t$ would be:

$$F_{intrct}(t) = F^i(\tau = t, t)$$  \hspace{1cm} (32)

Fig. 15a shows the above procedure for the same human force ($F_H$) as we studied in the previous section. The prediction of human force for $t \geq 1.14$, $F_H$, is shown with blue solid line and the boundary values at $t_m = 1.5$ and $t_f = 2.8$ are marked as magenta dots. The PM model is employed and the predicted interaction force, $\hat{F}^i(\tau, t)$, is calculated (green dashed line). The estimated interaction force at time $t$, $F_{intrct}(t)$, is obtained and marked by a red star. Note that the estimated value is very different from the offline calculated interaction force, $F^i(t)$ (red dashed-dotted line). However, as time increases, $\hat{F}^i$ approaches $F^i$ and the estimated value $F_{intrct}(t)$ gets closer to $F^i(t)$. Fig. 15b shows the evolution of $\hat{F}^i$ with several intermediate curves. As $\hat{F}^i$ curves approach $F^i$, the estimated interaction force values (red stars) get closer to the offline calculated interaction force.

Fig. 16 compares the performance of the online and the offline controllers. Fig. 16a compares the estimated interaction force with the offline-calculated one. Note that the estimated force is the same as the red star marks in Fig. 15b. The motion trajectories of both online and offline controllers are also compared with the human-robot motion trajectory ($x_{HR}$) using polynomial model (PM) and estimated interaction force (Est).

**Remark**

In addition to the simplicity and high performance, the proposed estimator possesses the following interesting features:

1) The proposed estimator guarantees the smoothness of
the estimated interaction force (subject to the smooth-ness of the human applied force)

2) The boundary conditions are always satisfied: 
\( F_{\text{intrct}}(t_i) = F_H(t_i), \ F_{\text{intrct}}(t_m) = F_H(t_m) \) and 
\( F_{\text{intrct}}(t_f) = F_H(t_f) \). Therefore, as \( t \to t_f \), we have 
\( \tilde{F}(\tau, t) \to F_i(\tau) \)

B. Statistical Evaluation

We are implementing the control strategy on a 4 DOF Barrett’s WAM robotic arm [46] as our future work. Thus, to make sure that the dynamics of the robot and the object will be modeled precisely during our simulations, we used MathWorks’ SimMechanics simulation environment and the STL files for the WAM arm (provided by the manufacturer). To be faithful to the experimental setup, we used a pot as to be manipulated object; with the same kinematic and dynamic properties as the actual pot. Fig. 17 shows the simulation setup for the controller evaluation.

Since the robot has only 4 DOF, we use a passive ball joint between the robot and the pot. Therefore, the orientation of the pot is controlled only by the human. In the simulation, we used the orientation data that was recorded from IMU during the experiment to reproduce the correct orientation of the pot.

Similar to the implementation in Sec. VII-A, we assumed that the robot has a full knowledge about the task. Also, we assumed that no friction or non-linearity exists in the robot and it can be controlled perfectly. We implemented the control diagram in Fig. 14 as the robot controller. The desired trajectory \( x_d \) is a minimum-jerk trajectory, given by (1). The impedance controller follows (27) and the “Interaction Model” estimator is described by (30)-(32). To measure the performance of the online controller, the same scenario as in Sec. VII-A was followed. To evaluate the performance of the online controller, we took the same approach as we discussed in Sec. VII-C and utilized the same RMSE measure as in (29). Let “Est” refer to the population of RMSE error samples obtained in this scenario. The mean and standard deviation for Est is \( (2.43 \pm 2.37) \) cm.

Consider Fig. 16a; while the estimated interaction force \( F_{\text{intrct}} \) follows the offline-calculated interaction force \( F_i \), there is a considerable difference between them. Also recall the poor performance of the controller when no information about the interaction force is available (discussed in Sec. VII-C). These observations suggest that the performance of the proposed online controller (using the estimator) would be higher than ZF model, but probably lower than PM model. To study this hypothesis, we set up another repeated measures ANOVA test. The ANOVA model has the same factors as the one described in Sec. VII-C. Except that, the within-subjects factor “model” has five levels (PM, VL, ME, ZF, Est) here. The test includes a total of 15 (3 × 5) repeated measurements for each of the 25 subjects.

Mauchly’s test of sphericity showed that the assumption of sphericity had been violated (\( \chi^2(104) = 1154, p < 0.0001 \)), and therefore, a Greenhouse-Geisser correction (\( \hat{\epsilon} = 0.18648 \)) was applied. The repeated measures ANOVA with a Greenhouse-Geisser correction determined that the mean RMSE differed statistically significantly among different models (\( F(0.7459, 17.1562) = 25.497, p < 0.0001 \)). Furthermore, no statistically significant interaction was observed between “model” and other factors (\( p > 0.15 \) for all). The significance level was \( \alpha = 5\% \) in the test. A pairwise multiple comparisons post-hoc test with the Bonferroni correction was performed. We observed that the mean RMSE error is statistically significantly higher when ZF is used (\( p < 0.0005 \) for all four pairwise tests). Moreover, no statistically significant difference was observed between the means of PM, VL, ME and Est (\( p > 0.61 \) for all six tests).

The test results supports our hypothesis in part. That is, the estimator provides enough information about the interaction, that the performance of the controller is statistically significantly higher than ZF model. However, no enough evidence existed to suggest that the performances of online controller (Est) is statistically significantly different from offline controllers (with PM, VL and ME models). These observations suggest that the human-robot cooperation strategy, proposed by the control scheme in Fig. 14, would be a promising candidate for an effective human-robot cooperative manipulation.

C. Discussion

A key limitation of our simulation approach is that the human applied forces had been recorded during the experiment (between humans) and then, they were played back during simulations (between human and robot). In other words, we did not model the possible changes in the human applied forces due to the robot’s actions in our simulations. In fact, we do not have access to a model that fully describes how human behavior varies in response to an external force. As a result, although the simulation results are very encouraging, unsatisfactory interactions may appear during an actual human-robot interaction.

On the other hand, human perception is usually noisy. So, if the robot’s applied forces are in the range that human expects to perceive, it may not influence human’s behavior.
Based on the promising results observed in the simulations, we speculate that the online controller will demonstrate a satisfactory performance in practice as well. But to test this hypothesis, one needs to setup a new study and test the online controller in a real human-robot experiment. We explain our future work on this matter in Sec. IX.

IX. Conclusion and Future Works

In this paper we studied the properties of the exchanged force during a dyadic-hand manipulation task. Using the existing theories of human-hand motions, we proposed a polynomial model (PM) for the interaction force in a dyadic reaching movement. To validate our model, we conducted a study with 22 subjects. We compared our model with the virtual linkage (VL) model and minimum-energy (ME) model in a 5-D feature space and observed the followings: First, the features of the interaction force are statistically significantly different among the models (except between ME and PM in dyadic mode). Second, while PM model would effectively capture the difference between the bimanual and dyadic types of collaboration, VL would always fail to do so and no enough evidence existed that ME can do the same.

Next, we embedded the models in an impedance controller and observed the followings: First, the performance of the controller is statistically significantly higher when the interaction force information is provided to the robot (using any model). Second, no enough evidence existed to suggest that the performances of different controllers (using all models) are statistically significantly different from each other. We concluded that the proposed model captures the critical information conveyed by the interaction force and provides a low-order approximation for it.

To be able to embed the interaction model in an online controller, the model should be causal and should depend only on the human’s measured force. By applying the polynomial model on a prediction of human’s future forces, we proposed a causal estimator for the interaction force. We showed that the proposed online controller demonstrates a high performance, similar to the offline controllers. That is, no enough evidence existed to suggest that their performances are statistically significantly different. Considering 1) the difference between the estimated and the offline-calculated versions of the interaction force and 2) the poor performance of the controller when no interaction force is provided, we concluded that these encouraging results are due to an interplay between the impedance controller and the interaction model. Thus, we speculated that the online controller would be a promising candidate for an effective human-robot cooperative interaction.

All these promising results have been achieved by applying our knowledge of the task to the control problem. For instance, in our case, a computational model for the reaching movement is the core of the proposed controller and estimator. While the task information is shared between the cooperating agents, the parameters of the task might not be. For instance, in dyadic reaching movement, the initial and final configurations ($x_i$ and $x_f$) are known for both persons, but they may have different preferred speeds (different $t_m$ and $t_f$). The learning trials are used to let the agents negotiate on these parameters and come to an agreement about the value of these parameters.

Similarly, the robot needs to learn about the human’s preferred values for these parameters. In case of the reaching movement, the robot needs to know the values of $t_i$, $t_m$, and $t_f$ for both generating its desired trajectory ($x_d$) and estimating the interaction force (see sections VII-A and VIII-A). In the simulations, we used the recorded data to provide these information to the robot. For our future experiment with the WAM arm, we will first let the human performs the task in the single-person-bimanual mode. Next, the parameters will be obtained, using the person’s recorded data. Then, human and the robot will be given few learning trials to adapt to each other. We speculate that the human satisfaction in this scenario would be as high as when the task is performed with a human partner. Designing an adaptation algorithm for these parameters or an online learning mechanism, using Bayesian approach for instance, would be an interesting extension to this work.

Appendix

To be able to quantitatively assess different aspects of the signals appearing in the interaction, we employ five performance metrics as follows. A complete list of related indexes and their interpretations (according to the associated human assessment) has been presented in [23]. The variables used in defining the following indexes (such as $\alpha$, $F_{sum}$ and effective forces $f_1^*$ and $f_2^*$) are defined in Sec. III-D. Also, $t_i$ and $t_f$ are the start time and the end time of the cooperative manipulation.

**Average Wasted Effort:**

The index measures the level of disagreement between the applied forces. It is defined as:

$$I_1 = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} M_1(t) \, dt$$  \hspace{1cm} (33)$$

where

$$M_1(t) = \frac{\|F_{sum}(t)\|}{\|f_1^*(t)\| + \|f_2^*(t)\|}$$

Since $F_{sum} = f_1^* + f_2^*$, a higher value of the index indicates less average wasted efforts. The index is suitably bounded between zero and one ($0 \leq I_1 \leq 1$), see [23].

**Energy-wise Similarity:**

The index measures the similarity between the effective forces in terms of the signal-energy. It is defined as:

$$I_2 = 1 - \left|\frac{N_1 - N_2}{N_{sum}}\right|$$  \hspace{1cm} (34)$$

where

$$\begin{cases} N_1 = \int_{t_i}^{t_f} \|f_1^*(t)\| \, dt \\ N_2 = \int_{t_i}^{t_f} \|f_2^*(t)\| \, dt \\ N_{sum} = \int_{t_i}^{t_f} \|F_{sum}(t)\| \, dt \end{cases}$$

**Energy-wise Similarity:**

$$\begin{cases} I_1 = \int_{t_i}^{t_f} \|f_1^*(t)\| \, dt \\ I_2 = \int_{t_i}^{t_f} \|f_2^*(t)\| \, dt \\ I_{sum} = \int_{t_i}^{t_f} \|F_{sum}(t)\| \, dt \end{cases}$$
A higher value of the index indicates smaller differences between the contributions of the subjects (in terms of signal-energy of the forces). The index is suitably bounded between zero and one (0 ≤ I ≤ 1), see [23].

**Time-wise Similarity:**

The index measures the similarity between the effective forces at each point in time. It is defined as:

\[
I_t = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} M_3(t) \, dt
\]  

where

\[
M_3(t) = 1 - \frac{\|f^*_1(t)\| - \|f^*_2(t)\|}{\|F_{sum}(t)\|}
\]

A higher value of the index indicates more symmetry between the effective forces. The index is suitably bounded between zero and one (0 ≤ I ≤ 1), see [23].

**Average of α(t):**

The index measures the average value of α during the task, and is related to the average of the interaction force. It is defined as:

\[
I_4 = 1 - \frac{|N_α|}{\mathcal{M}_α}
\]

where

\[
N_α = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} (\alpha(t) - \frac{1}{2}) \, dt
\]

and \(\mathcal{M}_α\) is the maximum value of \(|N_α|\) for a given set of signals \(α(t)\). A higher value of the index is associated with the average value of \(α\) closer to 0.5. The index is bounded between zero and one by definition (0 ≤ I ≤ 1).

**Average Rate of α(t):**

The index measures the average rate of the variations of α. It is defined as:

\[
I_5 = 1 - \frac{N_r}{\mathcal{M}_r}
\]

where

\[
N_r = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} \|\alpha(t)\| \, dt
\]

and \(\mathcal{M}_r\) is the maximum value of \(N_r\) for a given set of signals \(α(t)\). A higher value of the index indicates less average variations in \(α\). For instance, \(I_5 = 1\) holds only if \(α(t) = \text{const}\). The index is bounded between zero and one by definition (0 ≤ I ≤ 1).

**References**


